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Mathematics Syllabus 5 Periods – S6-S7

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[1] The syllabus has undergone minor changes, including clarifications, rewording, error corrections and harmonisation between the language versions.

European Schools Mathematics Syllabus Year S6&7 5 Periods

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1. General Objectives

The European Schools have the two objectives of providing formal education and of encouraging pupils' personal development in a wider social and cultural context. Formal education involves the acquisition of competences (knowledge, skills and attitudes) across a range of domains. Personal development takes place in a variety of spiritual, moral, social and cultural contexts. It involves an awareness of appropriate behaviour, an understanding of the environment in which pupils live, and a development of their individual identity.

These two objectives are nurtured in the context of an enhanced awareness of the richness of European culture. Awareness and experience of a shared European life should lead pupils towards a greater respect for the traditions of each individual country and region in Europe, while developing and preserving their own national identities.

The pupils of the European Schools are future citizens of Europe and the world. As such, they need a range of competences if they are to meet the challenges of a rapidly changing world. In 2006 the European Council and European Parliament adopted a European Framework for Key Competences for Lifelong Learning. It identifies eight key competences which all individuals need for personal fulfilment and development, for active citizenship, for social inclusion and for employment:

1. Literacy competence
2. Multilingual competence
3. Mathematical competence and competence in science, technology and engineering
4. Digital competence
5. Personal, social and learning to learn competence
6. Citizenship competence
7. Entrepreneurship competence
8. Cultural awareness and expression competence

The European Schools' syllabuses seek to develop all of these key competences in the pupils.

Key competences are that general, that we do not mention them all the time in the Science and Mathematics syllabuses.

2. Didactical Principles

General

In the description of the learning objectives, competences, connected to content, play an important role. This position in the learning objectives reflects the importance of competences acquisition in actual education. Exploratory activities by pupils support this acquisition of competences, such as in experimenting, designing, searching for explanations and discussing with peers and teachers. In science education, a teaching approach is recommended that helps pupils to get acquainted with concepts by having them observe, investigate and explain phenomena, followed by the step to have them make abstractions and models. In mathematics education, investigations, making abstractions and modelling are equally important. In these approaches, it is essential that a maximum of activity by pupils themselves is stimulated – not to be confused with an absent teacher: teacher guidance is an essential contribution to targeted stimulation of pupils' activities.

Mathematics

Careful thought has been given to the content and the structure to where topics are first met in a pupil’s time learning mathematics in secondary education. It is believed that this is a journey and if too much content is met at one point, there is a risk that it will not be adequately understood and thus a general mathematical concept will not be fully appreciated. By limiting the content of this syllabus (found in section 4.2.) each year more time can be used to develop core mathematical concepts that may have been met before or new mathematical concepts introduced are given ample time for extension. It must be noted that extension activities are conducted at the discretion of the teacher, however, it is suggested that rather than look at a vertical approach to extension a horizontal approach is used, thus giving the pupil a deeper understanding of the mathematical concept (in section 4 the word ‘limitation’ is used to ensure the extension does not go too far).

Furthermore, to this point it is believed that with a focus on competences this syllabus can encourage pupils to have a greater enjoyment of mathematics, as they not only understand the content better but understand the historical context (where it is expected a history of mathematics can be told over the cycles) and how the mathematics can be applied in other subjects, cross cutting (these can be seen in the fourth column in section 4.2.). As such the syllabuses have specifically been designed with reflection to the key competences (section 1) and the subject specific competences (section 3.1). In some cases, the key competences are clear for example the numerous history suggested activities (shown by the icon ) that maps to key competency 8 (Cultural awareness and expression). In other areas the link may not be so apparent.

One of the tasks in the pupil’s learning process is developing inference skills, analytical skills and strategic thinking, which are linked to both the key and subject specific competences. This is the ability to plan further steps in order to succeed solving a problem as well as dividing the process of solving more complex problems into smaller steps. A goal of teaching mathematics is to develop pupil’s intuitions in mathematics appropriate for their age. The ability to understand and use mathematical concepts (e.g., angle, length, area, formulae and equations) is much more important than memorising formal definitions.

This syllabus has also been written so that it can be accessible by teachers, parents and pupils. This is one reason why icons have been used (listed in section 4.2.). These icons represent different areas of mathematics and are not necessarily connected to just one competency but can cover a number of competences.

To ensure pupils have a good understanding of the mathematics the courses from S1 to S7 have been developed linearly with each year the work from the previous year is used as a foundation to build onto. Thus, it is essential before commencing a year the preceding course must have been covered or a course that is similar. The teacher is in the best position to understand the specific needs of the class and before beginning a particular topic it is expected that pupils have the pre-required knowledge. A refresh is always a good idea when meeting a concept for the first time in a while. It should be noted that revision is not included in the syllabus, however, as mentioned earlier about limiting new content, there is time to do this when needed.

The use of technology and digital tools plays an important role in both theoretical and applied mathematics, which is reflected in this syllabus. The pupils should get the opportunity to work and solve problems with different tools such as spreadsheets, computer algebra system (CAS) software, dynamic geometric software (DGS), programming software or other software that are available in the respective schools. Technology and digital tools should be used to support and promote pupils' understanding, for example by visualising difficult concepts and providing interactive and personalised learning opportunities, rather than as a substitute for understanding. Their use will also lead to improved digital competence.

Teachers have full discretion with how to teach this course, materials to use and even the sequence the content is taught in. The content and the competencies (indicated in the tables in section 4.2., columns 2 and 3) to be covered is, however, mandatory.

The S6 5 Period Course

This course has been specifically written for those that are wishing to choose to study mathematics at a higher level or those subjects that require higher level mathematics. It is also a course for pupils that love mathematics and enjoy going deeper into the theory.

Pupils must note that the 4 Period and 6 Period courses in S5 are different. Thus, pupils that have not studied the 6 Period course in S5 may find themselves at a disadvantage, as this 5 Period course is a natural continuation to the work done in the 6 Period course of S5. It is certainly recommended that pupils have a good understanding of the topics covered in the 6 Period course of S4 and S5.

The S7 5 Period Course

This course is a continuation from the S6 5 Period course, and it is strongly advised that anyone wishing to start this course has a good understanding of the topics and competences covered in the S6 5 Period course.

3. Learning Objectives

3.1. Competences

The following are the list of subject specific competences for mathematics. Here the key vocabulary is listed so that when it comes to reading the tables in section 4.2. the competency being assessed can be quickly seen. Please note that the list of key vocabulary is not exhaustive, and the same word can apply to more than one competency depending on the context.

Further information about assessing the level of competences can be found in section 5.1. Attainment Descriptors. The key concepts here are those needed to attain a sufficient mark.

	Competency	Key concepts (attain 5.0-5.9)	Key vocabulary
1.	**Knowledge and comprehension**	Demonstrates satisfactory knowledge and understanding of straightforward mathematical terms, symbols and principles	Apply, classify, compare, convert, define, determine, expand, factorise, identify, know, manipulate, name, order, prove, recall, recognise, round, simplify, understand, verify
2.	**Methods**	Carries out mathematical processes in straightforward contexts, but with some errors	Apply, calculate, construct, convert, draw, manipulate model, plot, simplify sketch solve, use, verify
3.	**Problem solving**	Translates routine problems into mathematical symbols and attempts to reason to a result	Classify, compare, create, develop, display, estimate, generate, interpret, investigate, measure, model, represent, round, simplify, solve
4.	**Interpretation**	Attempts to draw conclusions from information and shows limited understanding of the reasonableness of results	Calculate, conduct, create, develop, discover, display, generate, interpret, investigate, model
5.	**Communication**	Generally presents reasoning and results adequately using some mathematical terminology and notation	Calculate, conduct, create, discover, display, interpret, investigate, model, present
6.	**Digital competence**	Uses technology satisfactorily in straightforward situations	Calculate, construct, create, display, draw, model, plot, present, solve

3.2. Cross-cutting concepts

Cross cutting concepts will be carried by the joint competences. The list of cross cutting concepts that will be composed will be shared by all science and mathematics syllabuses. The tentative list to be taught is based on the next generation science standards in the United states (National Research Council, 2013):

	Concept	Description
1.	**Patterns**	Observed patterns of forms and events guide organisation and classification, and they prompt questions about relationships and the factors that influence them.
2.	**Cause and effect**	Mechanism and explanation. Events have causes, sometimes simple, sometimes multifaceted. A major activity of science is investigating and explaining causal relationships and the mechanisms by which they are mediated. Such mechanisms can then be tested across given contexts and used to predict and explain events in new contexts.
3.	**Scale, proportion and quantity**	In considering phenomena, it is critical to recognise what is relevant at different measures of size, time, and energy and to recognise how changes in scale, proportion, or quantity affect a system's structure or performance.
4.	**Systems and system models**	Defining the system under study—specifying its boundaries and making explicit a model of that system—provides tools for understanding the world. Often, systems can be divided into subsystems and systems can be combined into larger systems depending on the question of interest
5.	**Flows, cycles and conservation**	Tracking fluxes of energy and matter into, out of, and within systems helps one understand the systems' possibilities and limitations.
6.	**Structure and function**	The way in which an object or living thing is shaped and its substructure determine many of its properties and functions and vice versa.
7.	**Stability and change**	For natural and built systems alike, conditions of stability and determinants of rates of change or evolution of a system are critical for its behaviour and therefore worth studying.
8.	**Nature of Science**	All science relies on a number of basic concepts, like the necessity of empirical proof and the process of peer review.
9.	**Value thinking**	Values thinking involves concepts of justice, equity, social–ecological integrity and ethics within the application of scientific knowledge.

In the mathematics syllabuses, the concepts 5 and 8 will be addressed only to a limited extent.

The lists of competences and cross cutting concepts will serve as a main cross-curricular binding mechanism. The subtopics within the individual syllabuses will refer to these two aspects by linking to them in the learning goals.

4. Content

4.1. Topics

This section contains the tables with the learning objectives and the mandatory content for the strand Mathematics in S6 and S7 (5 periods per week).

4.2. Tables

How to read the tables on the following pages

The learning objectives are the curriculum goals. They are described in the third column. These include the key vocabulary, highlighted in bold, that are linked to the specific mathematics competences found in section 3.1. of this document. These goals are related to content and to competences. The mandatory content is described in the second column. The final column is used for suggested activities, key contexts and phenomena. The teacher is free to use these suggestions or use their own providing that the learning objective and competencies have been met. Please note that the word ‘limitation’ is used to ensure that when extension is planned it is planned with the idea of horizontal extension rather than vertical extension as mentioned in section 2. of this document.

Use of icons

Furthermore, there are six different icons which indicate the areas met in the final column:

	Activity
	Cross-cutting concepts
	Digital competence
	Extension
	History
	Phenomenon

Each of these icons highlight a different area and are used to make the syllabus easier to read. These areas are based on the key competences mentioned in section 1 of this document.

S6 5 Period (5P)

Year 6 (5P)	TOPIC: ALGEBRA		
Subtopic	**Content**	**Learning objectives**	**Key contexts, phenomena and activities**
Introduction to complex numbers	Complex number of the form $z = a + \mathrm{i}b$ Equations with complex solutions	**Determine** the real part, imaginary part, complex conjugate and the inverse of a complex number **Calculate** with complex numbers: sum, product, division **Solve** a quadratic equation with real coefficients and complex solutions, i.e.: $az^2 + bz + c = 0$ with $a, b, c \in \mathbb{R}$, $a \neq 0$ et $\Delta < 0$ **Investigate** equations involving complex numbers *Limitation: only up to and including second degree equations with real coefficients.*	Consider the historical context of complex numbers, for example: • Resolution of the third-degree equation: how Bombelli extended Cardano's method for solving equations $x^3 = px + q$ • $\sqrt{-1}$ is used to calculate real solutions to a system of equations • Basel problem leading to Riemann's hypothesis: $\sum_{n=1}^{\infty} \frac{1}{n^2} = 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \cdots = \frac{\pi^2}{6}$ $\varsigma(z) = \sum_{n=1}^{\infty} \frac{1}{n^z} = \frac{1}{1^z} + \frac{1}{2^z} + \frac{1}{3^z} + \cdots$ $\varsigma(2) = \frac{\pi^2}{6}$
Sequences	Concept of sequences	**Understand** the concept of sequences starting from examples	Explore concepts of growth and decay. • Project suggestion: Study growth and decay of natural, scientific and economic phenomena. • Pupils can explore: ○ The Collatz sequence ○ Recamán's sequence ○ The prime numbers ○ The Mersenne primes ○ The Fibonacci sequence and golden ratio

Year 6 (5P)	TOPIC: ALGEBRA		
Subtopic	**Content**	**Learning objectives**	**Key contexts, phenomena and activities**
			Unsolved problem in mathematics: Does the Collatz sequence eventually reach 1 for all positive integer initial values?
			Design an algorithm to investigate the total stopping time (i.e. the smallest I such that a_i is 1) depending on the starting value.
	The concept defining a sequence explicitly or recursively	**Know** the notation of explicitly or recursively defined sequences (first term u_0 or u_1)	
		Calculate, with and without a technological tool, terms of a sequence defined explicitly or recursively	
	Graphical behaviour of sequences	**Observe** the graphical behaviour of a sequence	
	Increasing, decreasing sequences	**Determine** whether a sequence defined explicitly is increasing or decreasing (monotonic)	Determine whether a sequence defined by recurrence is increasing or decreasing.
	Limits	**Calculate** the limit of a sequence defined explicitly	Discuss the following sequences: $(-1)^n$, $\frac{(-1)^n}{n}$, $\frac{1}{n}$.
		Use a technological tool to enter a sequence both explicit and recurrence relation and interpret its properties	

S6 5 Period (5P)

Note 1:

Except where specified, all the knowledge and skills given in the analysis section apply without a technological tool to only this set of basic functions for a, b, c, $\lambda \in \mathbb{R}$, $\lambda \neq 0$:

- polynomial functions of degree ≤ 3
- $\frac{a\cdot x+b}{c\cdot x+d}$
- $\sqrt{a \cdot x + b}$
- $\lambda \cdot \cos(a \cdot x + b) + c$
- $\lambda \cdot \sin(a \cdot x + b) + c$
- $\tan x$
- $\lambda \cdot \alpha^x$ for $\alpha \in \mathbb{R}_{>0}$, $\alpha \neq 1$
- $\lambda \cdot \mathrm{e}^{a\cdot x+b}$
- $\lambda \cdot \log_\alpha(a \cdot x + b) + c$ for $\alpha \in \mathbb{R}_{>0}$, $\alpha \neq 1$, especially the particular cases where $\alpha = \mathrm{e}$ (ln), $\alpha = 10$ (log) and $\alpha = 2$ (lb)

Note 2:

The use of technology throughout this section will enable the study of analysis to not just be limited to the above functions.

Note 3:

It is recommended that the work on analysis is split over two semesters, leaving exponentials and logarithms until the second semester. This allows a revision of the concepts studied earlier in the year, applied to the new functions.

Year 6 (5P)	**TOPIC: ANALYSIS**			
Subtopic	**Content**	**Learning objectives**	**Key contexts, phenomena and activities**	
Introduction to real functions	Functions as models	**Review** the concept that real life situations can be modelled mathematically (using functions, graphs, …)		
	Properties of a function	**Understand** the ideas of domain and range of a function **Explore** the concepts of parity and periodicity of a function		

Year 6 (5P)	TOPIC: ANALYSIS			
Subtopic	**Content**	**Learning objectives**	**Key contexts, phenomena and activities**	
		Determine the domain, the range, possible zeros, sign, parity (even, odd functions, or neither) and periodicity of functions, using suitable methods, e.g. graphically, numerically or algebraically.		
	Combining functions	**Define** the sum, product, quotient and composition of functions **Investigate** the effect of these operations on the domain **Understand** the concept of an inverse function in relation with the composition of functions	 	Discuss removable discontinuities and continuity in general as well as left and right limits. Algebraic vs Transcendental numbers.
	Graphs of functions	**Sketch** the graphs of the given basic functions (see Note 1 above) **Explore** the behaviour of the graphs using a technological tool **Explore** the connections between the graphs of inverse functions **Investigate** the effect on the graph of a function of the following transformations, starting with the work in S5 on completing the square: $f(x)+k$, $f(x+k)$, $k\cdot f(x)$, $f(k\cdot x)$, $k\in\mathbb{R}$		
Limits	Limits of the basic functions	**Understand** the concepts of finite and infinite limits of a function tending to a point or infinity		Explore: • the concept of infinity • Zeno's paradoxes • Georg Cantor and different sizes of infinity. • the concept of continuity of a function from the right [resp. left] to a point

Year 6 (5P)	**TOPIC: ANALYSIS**			
Subtopic	**Content**	**Learning objectives**	**Key contexts, phenomena and activities**	
		Determine limits: at infinity, one sided limits (from the right and from the left), and limits at an end point of the domain		
Differentiation	Derivative of a function at a point (gradient)	**Understand** the significance of the derivative number of a function at a point (gradient)		Instantaneous speed, acceleration, and the jerk - fundamental relationship of the dynamics in mechanics.
		Explore the derivative of a function at a point from first principles, e.g., $\frac{\mathrm{d}y}{\mathrm{d}x} = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ for simple functions : • Polynomial functions degree ≤ 3 • $\frac{1}{x}$		Leibnitz and Newton controversy. **Explore** the derivative of the square root function $\sqrt{x}$ from first principles
		Apply the rules of differentiation to the set of basic functions (including sum, product, quotient and composition (chain) rules where appropriate)		
	First and second derivative functions	**Calculate** first and second derivative functions **Interpret** their relevance		Calculate higher order derivatives of functions.
	Tangent to a graph	**Calculate** the gradient of a tangent to a curve at a given point **Determine** the equation of a tangent at a given point		

Application of limits and derivatives	Behaviour of a function	**Apply** the concepts of limits and the process of differentiation with regard to: • finding vertical or horizontal asymptotes • where the function is increasing or decreasing • finding local and global extrema • where the function is convex or concave • finding points of inflexion **Apply** to optimisation problems **Explore** the characteristics of a function knowing the graph of its derivative and vice versa		Economics – marginal analysis.
Exponential and Logarithms	Logarithmic and exponential functions	**Review** the idea of exponential functions and **introduce** logarithmic functions **Explore** the properties of these functions and their graphs		
	Natural logarithm and the exponential function of base ***e***	**Investigate** the exponential function of base **e** as solution of $y' = y$ and $y(0) = 1$, and the natural logarithm function as an inverse of the exponential function	 	Differential equation of first and second order Describe Euler method for solving $y' = y$, design an algorithm and implement it for different step sizes

		Define the natural logarithmic and the exponential function of base e		
		Understand the properties of indices and use these to **review and extend** a pupil's understanding of the rules for logarithms: • $a^p \cdot a^q = a^{p+q}$ • $a^{-p} = \frac{1}{a^p}$ • $(a^p)^q = a^{p \cdot q}$ • $\log_a(u \cdot v) = \log_a u + \log_a v$ • $\log_a \left(\frac{1}{u}\right) = -\log_a u$ • $\log_a(u^n) = n \cdot \log_a u$ • where $a, b, u, v \in \mathbb{R}_{>0} \backslash \{1\}, p, q \in \mathbb{R}$		Music and Indices investigation: The note A below middle C vibrates at 220Hz. A# vibrates at $220x2^{\frac{1}{12}}$Hz. What about B, middle C, C# etc.?
	Equations and inequalities	**Solve** equations and inequalities involving logarithms and/or exponentials with or without base e, **discuss** when to reject an invalid solution		Napier/Briggs logarithm tables. Describe Newton's method (tangent method) for simple functions such as $\ln x + x$ and design an algorithm to find the root of the function.
	Properties of logarithmic and exponential functions	**Determine** domain, intersection with the coordinate axes, limits, asymptotes, derivative and how it may vary, tangent at a point, extrema, curvature and points of inflexion leading to sketching the graphs of the functions		Use applications from the fields of physics, biology, economics and others.

Year 6 (5P)	TOPIC: GEOMETRY		
Subtopic	**Content**	**Learning objectives**	**Key contexts, phenomena and activities**
Vectors model for 2D space: equations of straight lines and applications	Lines in the plane, points on a line	**Determine** and **represent** the vector equation of a straight line, the parametric equation of a straight line, the cartesian equation of a straight line	Represent the solutions of an equation: geometrical loci. How are mathematics and philosophy similar? “I think; therefore I am.” René Descartes the father of modern philosophy. Cartesian geometry.
	Distance between two points	**Calculate** the distance between two points	
	Equations of circles	**Determine** the equation of a circle, as the set of all points equidistant from a centre **Identify** the radius of a circle centred on the origin, given its equation	Identify the coordinates of the centre and the radius of a circle not centred on the origin, given its equation
	Intersection of lines	**Analyse** the relative position of two lines: intersecting lines, coincident lines, parallel lines **Calculate** the coordinates of the point of intersection between two lines	• Euclidian geometry. The History of the Parallel Postulate. • Euclidean, Elliptic and Hyperbolic spaces.
	Angle between two intersecting lines	**Understand** the scalar product of 2D vectors **Understand** the concept of orthogonal vectors **Calculate** the angle between two intersecting lines	
	Parallels and perpendicular lines	**Recognise** parallel lines from their equations **Determine** the equation of a line parallel to a given line, passing through a given point	

Year 6 (5P)	TOPIC: GEOMETRY			
Subtopic	**Content**	**Learning objectives**	**Key contexts, phenomena and activities**	
		Recognise perpendicular lines from their equations **Determine** the equation of a line perpendicular to a given line, passing through a given point		
	Perpendicular projection of a point on a line	**Define** the perpendicular projection of a point to a line **Calculate** the coordinates of the perpendicular projection of a point to a line		
	Distance between a point and a line, two parallel lines	**Analyse** the relative position of a point and a line **Define** the distance from a point to a line **Calculate** the distance from a point to a line **Calculate** the distance between parallel lines		Investigate the algebraic equation of a circle
	Applications of a line in a plane	**Find** the velocity vector of a moving object and the speed of the object		

<table>
<tr><th>Year 6 (5P)</th><th colspan="4">TOPIC: STATISTICS AND PROBABILITY</th></tr>
<tr><th>Subtopic</th><th>Content</th><th>Learning objectives</th><th colspan="2">Key contexts, phenomena and activities</th></tr>
<tr><td>Combinatorics</td><td>Tree diagrams</td><td>Represent events in tree diagrams</td><td></td><td>Discuss the limitations of the tree diagrams.</td></tr>
<tr><td></td><td>Permutations and combinations</td><td>Model situations of elementary combinatorial analysis:
<table>
<tr><th></th><th>Without repetition</th><th>With repetition</th></tr>
<tr><td>Permutations of n objects</td><td>$n!$</td><td></td></tr>
<tr><td>Permutations of k objects from n</td><td>$\frac{n!}{(n-k)!}$</td><td>n^k</td></tr>
<tr><td>Combinations</td><td>$\binom{n}{k}$
$=$
$\frac{n!}{k!\cdot(n-k)!}$</td><td>$\binom{n+k-1}{k}$
$=$
$\frac{(n+k-1)!}{k!\cdot(n-1)!}$</td></tr>
</table></td><td></td><td>Investigate the numbers of possible:
paths in grid problems
passwords
one-dimensional barcode (EAN protocol), ISBN and ISSN
phone and bank account numbers in different countries
number plates in different countries
book layouts on a shelf without or with constraint
ranking or podium place at a competition
lotteries in different
hands in card games
Write a simple program to calculate the faculty of a given integer number.

The Catalan numbers.</td></tr>
<tr><td></td><td>Combinational formulas</td><td>Apply the following combinational formulas:
$\binom{n}{0}=\binom{n}{n}=1$
$\binom{n}{1}=n$
$\binom{n}{k}=\binom{n}{n-k}$
$\binom{n}{k}=\binom{n-1}{k-1}+\binom{n-1}{k}$</td><td></td><td>Write a simple program to calculate the binomial coefficient of two given integer numbers and check the combinational formulas on the left with numerical examples.</td></tr>
</table>

Year 6 (5P)	TOPIC: STATISTICS AND PROBABILITY			
Subtopic	**Content**	**Learning objectives**	**Key contexts, phenomena and activities**	
	Pascal's triangle and Newton's binomial theorem	**Understand** Pascal's triangle **Use** Newton's binomial theorem: $(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} \cdot x^k \cdot y^{n-k}$		Prove and illustrate with Pascals' triangle: $\binom{n}{0} + \binom{n}{1} + \cdots + \binom{n}{n-1} + \binom{n}{n} = \sum_{k=0}^{n} \binom{n}{k} = 2^n$
	Sum notation	**Understand** how to write for finite and infinite sums using the sum notation, e.g., $\sum_{k=1}^{n} k$, $\sum_{k=1}^{n} k^2$, $\sum_{k=1}^{\infty} \frac{1}{k}$, $\sum_{k=1}^{\infty} \frac{1}{k^2}$		
Probability	Elementary probability	**Recall** the following general probability rules to **calculate** the probability of an event: • $P(\overline{A}) = 1 - P(A)$ • $P(A \cup B) = P(A) + P(B)$ for $A \cap B = \emptyset$ • $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ for $A \cap B \neq \emptyset$		• Use data collected from the class, e.g., late or on time to school and transportation mode used to go to school (car, bus, bicycle, ...). • Venn diagrams may be used to visualise these rules.
	Conditional probability	**Know** the rule and notations for conditional probability: $P_B(A) = P(A\|B) = \frac{P(A \cap B)}{P(B)}$ and **apply** it in appropriate situations		
	Dependent and independent events	**Identify** independent events by using the following equivalent formulae: • $P(A \cap B) = P(A) \cdot P(B)$ • $P_B(A) = P(A\|B) = P(A)$ • $P_A(B) = P(B\|A) = P(B)$		• Tree diagrams and contingency tables may be used to visualise these rules. • Use data from (social) sciences.

	Total probability law and Bayes' theorem	**Understand** and **apply** the total probability formula: $\mathrm{P(B)} = \mathrm{P(A_1)} \cdot \mathrm{P(B\|A_1)} + \mathrm{P(A_2)} \cdot \mathrm{P(B\|A_2)}$ **Understand** and **apply** Bayes' theorem (for $k \in \{1,2\}$): $\mathrm{P(A_k\|B)} = \frac{\mathrm{P(A_k)} \cdot \mathrm{P(B\|A_k)}}{\mathrm{P(B)}}$ *Limitation: the formula* $\mathrm{P(A\|B)} = \frac{\mathrm{P(B\|A)} \cdot \mathrm{P(A)}}{\mathrm{P(B\|A)} \cdot \mathrm{P(A)} + \mathrm{P(B\|\bar{A})} \cdot \mathrm{P(\bar{A})}}$ *must not expressly be known but, with appropriate indications, situations referring to it can be studied*		Investigate about vaccination and disease testing, blood testing of other type of tests: • Use Bayes' theorem to show that the reliability of a test depends on the percentage of infected people within a population • Let x be the proportion of infected people. Study the rational function $f(x) = \mathrm{P(infected\|positive)}$: increasing/decreasing, bounds … • To reduce the ratio of false positives, should you improve the sensitiveness or the specificity of the test?

Discrete distributions	Discrete random variables
	
	Probability density function
		Explain the concept of a finite discrete random variable and its probabilities
		
		Understand and **apply** the probability density function of a discrete random variable X, defined by
		$f(x_i) = \mathrm{P}(\mathrm{X} = x_i)$ for $1 \leq i \leq n$
				Random processes in daily life and in scientific contexts.
				
				Explore examples of discrete uniform distributions:
				• Flipping coins
				• Rolling dice
				Explore examples of discrete distributions:
				• Poisson distribution:
				  o Number of traffic accidents in one day
				  o Emission of alpha particles from a radioactive sample
				  o Number of misprints per page in books
				• Benford's law of anomalous numbers

	Cumulative distribution function	**Know** that the probabilities sum up to 1 **Understand** and **apply** the cumulative distribution function of a discrete random variable X, defined by: $\mathrm{F}(x) = \mathrm{P}(\mathrm{X} \leq x) = \sum_{x_i \leq x} \mathrm{P}(\mathrm{X} = x_i)$		
	Expected value	**Understand, interpret,** and **calculate** the expected value of a discrete random variable as $\mathrm{E}(\mathrm{X}) = \sum_k x_k \cdot \mathrm{P}(\mathrm{X} = x_k)$		Investigate the winnings expectation value for gambling games.
	Variance and standard deviation	**Understand, interpret,** and **calculate** the variance of a discrete random variable as $\mathrm{Var}(\mathrm{X}) = \sum_k \left(x_k - \mathrm{E}(\mathrm{X})\right)^2 \cdot \mathrm{P}(\mathrm{X} = x_k)$		Compare different games/insurances using expected value and determine if they are fair.
		Derive and **use** the formulae • $\mathrm{E}(\mathrm{X}^2) = \sum_k {x_k}^2 \cdot \mathrm{P}(\mathrm{X} = x_k)$ • $\mathrm{Var}(\mathrm{X}) = \mathrm{E}(\mathrm{X}^2) - (\mathrm{E}(\mathrm{X}))^2$ • $\sigma(\mathrm{X}) = \sqrt{\mathrm{Var}(\mathrm{X})}$		• Proof that for a random variable *X* that has a uniform distribution over $\{1, 2, \ldots, n\}$: $\mathrm{E}(\mathrm{X}) = \frac{n+1}{2}$ and $\mathrm{Var}(\mathrm{X}) = \frac{n^2-1}{12}$. • Explore linear combinations of random variable.
	Bernoulli trial and Binomial distribution	**Recognise** a Bernoulli trial and **explain** the conditions under which a random variable follows a binomial distribution		
		Calculate probabilities $\mathrm{P}(\mathrm{X} = \mathrm{k})$, $\mathrm{P}(\mathrm{X} \leq \mathrm{k})$, $\mathrm{P}(\mathrm{X} \geq \mathrm{k})$ and $\mathrm{P}(\mathrm{k} \leq \mathrm{X} \leq \mathrm{k}')$ for a random variable X with a binomial distribution: • by hand for $n \leq 4$ using the Bernoulli formula: $\mathrm{P}(\mathrm{X} = \mathrm{k}) = \binom{n}{k} \cdot p^k \cdot (1-p)^{n-k}$ • with a technological tool		Investigate with a technological tool how varying the parameters effect the symmetry of the graph of the binomial distribution.

	Binomial parameters, expected value and variance	**Understand** and **interpret** the concepts of expected value and variance of a random variable X with a binomial distribution in terms of its parameters p (success probability) and n (number of trials): • $\mathrm{E(X)} = n \cdot p$ • $\mathrm{Var(X)} = n \cdot p \cdot (1 - p)$		• Investigate under which conditions different distributions give the same expected value. • Poisson distribution as suitable for the number of specific events happening in a given time period, and as an approximation of a binomial distribution for “big” values of n and “small” values of p (e.g., $n > 50$ and $p < 0.1$).
	Modelling with the binomial distribution	**Recognise** and **model** situations where binomial distribution can be used		Explore examples of binomial distributions: • Galton board • Quality control of a product (with and without defect) • Multiple drawing from an urn with putting back • Probability of a certain number of boys/girls in a family with several children
	Binomial approximation	**Apply** the binomial distribution as an approximation when a small sample is taken by making successive draws from a large population *Limitation: just explain the idea, no proof needed*		

S7 5 Period (5P)

Year 7 (5P)	**TOPIC: ALGEBRA**		
Subtopic	**Content**	**Learning objectives**	**Key contexts, phenomena and activities**
Complex numbers	Geometric representation	**Represent** a complex number geometrically	Example from physics: complex representation of electrical quantities.
	Modulus and argument	**Determine** the modulus and argument of a complex number and its inverse	
		Determine the modulus and argument of the product and quotient of complex numbers	Design an algorithm that converts a complex number in exponential form.
	Different forms of complex numbers	**Use** the three different forms of complex numbers: • $x + \mathrm{i} \cdot y$, x and y real numbers • $r \cdot (\cos\theta + \mathrm{i} \cdot \sin\theta)$ where r is a positive number and $\theta \in]-\pi\,;\,\pi]$ • $r \cdot \mathrm{e}^{\mathrm{i}\theta}$ **Convert** from one form to another	• Discuss the consistency of the exponential notation: Can we prove that: ○ $\mathrm{e}^{\mathrm{i}\theta_1} \times \mathrm{e}^{\mathrm{i}\theta_2} = \mathrm{e}^{\mathrm{i}(\theta_1+\theta_2)}$? ○ $\left(\mathrm{e}^{\mathrm{i}\theta}\right)^n = \mathrm{e}^{\mathrm{i}n\theta}$? ○ $\frac{\mathrm{e}^{\mathrm{i}\theta_1}}{\mathrm{e}^{\mathrm{i}\theta_2}} = \mathrm{e}^{\mathrm{i}(\theta_1-\theta_2)}$? • Calculating the exact values of trigonometric ratios. • Proofs double angles formulae. • Can we define a complex logarithm? • Assuming complex functions can be differentiated: prove Euler’s identity.
		Investigate which of the above forms are best adapted to solve problems such as: • expressing the modulus and argument of a quotient • finding a locus • solving equations with indices	Discuss why $\mathrm{e}^{\mathrm{i}\pi} + 1 = 0$ is considered one of the most beautiful equations in mathematics.

	n^{th} roots of a complex number	**Solve** equations of the type $z^n = a$ with $a \in \mathbb{C}$, $n \in \mathbb{N}\backslash\{0;1\}$, and represent the solutions graphically **Check** calculations and solutions using a technological tool		
Sequences: arithmetic and geometric sequences	Arithmetic and geometric sequences	**Give** definitions of arithmetic and geometric sequences **Solve** problems involving the properties of arithmetic and geometric sequences: **finding** the first term, common difference, common ratio etc		Other problems to explore include: • human population • pandemics • cancer cells
	Limits of arithmetic and geometric sequences	**Explore** the limit of explicitly defined geometric and arithmetic sequences		
	Sum of consecutive terms	**Calculate** the sum of n consecutive terms of an arithmetic or geometric sequence **Use** sigma notation for sums of arithmetic and geometric sequences **Solve** problems given the sum **Use** the formulae for the n^{th} term and the sum of the first n terms of the sequence		Wheat and chessboard problem.
	Applications	**Solve** problems involving the properties of arithmetic and geometric sequences, e.g., compound and simple interest or growth and decay		

S7 5 Period (5P)

Note 1:

Except where specified, all the knowledge and skills given in this part of analysis section apply without a technological tool to only this set of basic functions for a, b, c, $\lambda \in \mathbb{R}$, $\lambda \neq 0$:

- polynomial functions of degree ≤ 3
- $\frac{P(x)}{Q(x)}$ where $P(x)$ and $Q(x)$ are polynomials functions of degree ≤ 2
- $\lambda \cdot \sqrt{a \cdot x + b} + c$
- $\lambda \cdot \cos(a \cdot x + b) + c$
- $\lambda \cdot \sin(a \cdot x + b) + c$
- $\tan x$
- $\lambda \cdot \mathrm{e}^{a \cdot x + b}$
- $\lambda \cdot (\mathrm{e}^{a \cdot x} + \mathrm{e}^{-a \cdot x})$
- $\lambda \cdot \ln(a \cdot x + b) + c$
- $\lambda \cdot x^{\alpha} \cdot \ln x$ for $\alpha \in \{-2, -1, 1, 2\}$

Note 2:

The use of technology throughout this section will enable the study of analysis to not just be limited to the above functions.

Year 7 (5P)	**TOPIC: ANALYSIS**		
Subtopic	**Content**	**Learning objectives**	**Key contexts, phenomena and activities**
Study of real functions		**Review** the concepts of functions already met in year 6 and apply them to the extended range of functions above	Properties of the Dirichlet function Describe the bisection method for simple functions such as $\mathrm{e}^x + x$ or $x^3 - x + 1$ and design an algorithm to find the root of the function.
	Indeterminate forms of limits	**Explore** indeterminate forms of limits such as "$\frac{\infty}{\infty}$", "$\frac{0}{0}$", "$0 \cdot \infty$" and **discuss** ways to determine them	The debate over the definition of 0^0.
	Oblique asymptotes	**Investigate** oblique asymptotes, possibly with the use of the technological tool **Determine** equations of oblique asymptotes (see Note 1 above)	

Year 7 (5P)	TOPIC: ANALYSIS		
Subtopic	**Content**	**Learning objectives**	**Key contexts, phenomena and activities**
Integration	Indefinite integral	**Understand** that the primitive (or antiderivative) of a function f is a function F whose derivative is equal to f, and that the indefinite integral is the set of all these functions	Mean speed theorem – Oxford Calculators of Merton College, Oresme and Galileo.
	Definite integral	**Know** that for a function f with antiderivative F and reals numbers a and b in the domain of f $$\int_a^b f(x)\mathrm{d}x = F(b) - F(a)$$ **Understand** that the definite integral gives the area bounded by the graph of a non-negative function f and the x-axis from $x = a$ to $x = b$ (for $a < b$)	The connection between integration and area can be explored algorithmically (for example using Python or GeoGebra).
	Integral properties	**Discover** the properties of integrals: if $a \leq b$ and $f \leq g$ on [a, b], then: • $\int_a^b f(x)\mathrm{d}x \leq \int_a^b g(x)\mathrm{d}x$ • $\int_a^b f(x)\mathrm{d}x = -\int_b^a f(x)\mathrm{d}x$ • $\int_a^b f(x)\mathrm{d}x + \int_b^c f(x)\mathrm{d}x = \int_a^c f(x)\mathrm{d}x$ • $\int_a^b \big(\lambda f(x) + \mu g(x)\big)\mathrm{d}x$ $= \lambda \int_a^b f(x)\mathrm{d}x + \mu \int_a^b g(x)\mathrm{d}x$	

		Calculate the integrals of the functions in note 1 above, possibly using integration by parts or integration by substitution (given the substitution) *Limitation: For $\frac{P(x)}{Q(x)}$: $P(x)$ and $Q(x)$ should be limited to linear functions*		
	Improper integral	**Understand** the concept of an improper integral **Calculate** improper integrals for simple cases (see Note 1)		
	Calculate areas and volumes of revolution	**Apply** integration to **calculate** areas in the plane and volumes of revolution about the x-axis		• Method of exhaustion to find the area of a circle. • Monte Carlo method for calculating the area under the curve. • Describe the following methods for estimating integrals and design an algorithm to implement them: ○ Brouncker’s method for calculating the area under the hyperbola ○ The trapezoidal rule ○ The rectangle rule • Discuss the speed of convergence and the accuracy of the above methods. Volume of a torus. Toricelli’s trumpet, finite volume but infinite surface area.
		Determine integrals using a technological tool		

Year 7 (5P)	TOPIC: GEOMETRY		
Subtopic	**Content**	**Learning objectives**	**Key contexts, phenomena and activities**
Vectors model for 3D spaces Lines and planes	Orthonormal system, points and unit vectors	**Know** what a coordinates system in a space is**,** given four non coplanar points $(\mathrm{O}; \mathrm{I}, \mathrm{J}, \mathrm{K})$ in a 3D space, and a point M in space, its coordinates (x, y, z) are defined by: $\overrightarrow{\mathrm{O}M} = x\overrightarrow{\mathrm{OI}} + y\overrightarrow{\mathrm{OJ}} + z\overrightarrow{\mathrm{OK}} = x\vec{\imath} + y\vec{\jmath} + z\vec{\mathrm{k}}$	• n-uplets of numbers as points in n-D spaces. • Spatial description of multidimensional phenomena, e.g., Quantum theory. Solve wolf, goat and cabbage problem using 3D geometry.
	Lines in two and three dimensions	**Define** a straight line with: • a vector and a point • two points **Know** the concept of vector equation of a line **Use**, for a line: • parametric equations • Cartesian equations	Movement in and above a 2D plane **–** navigation for boats and planes: tides, winds, course over water, course over ground, course to steer, leeway, …
	Parallelism and orthogonality in the 3D space	**Understand** and **use** parallel vectors in 3D space **Understand** and **use** the scalar product of two vectors in a 3D space**,** and perpendicular vectors	
	The vector product	**Calculate** the vector product of two vectors **Apply** the vector product to calculate areas of triangles and parallelograms	

		Use the scalar triple product to determine the volume of a parallelepiped **Conduct** the test for coplanar points		Volume of a tetrahedron.
	Planes in 3D space	**Define** a plane: • by two lines that lie in the plane • with two vectors and a point • with three points		• The Geometry of the Universe. • The history of Non-Euclidean geometry.
		Define a line as the intersection of two planes		Investigate the intersection of more than two planes (point defined as the intersection of three planes for example).
		Know the concept of a vector equation of a plane **Use**: • parametric equations of a plane • a Cartesian equation of a plane		
	Intersections in 3D space	**Determine** the intersection: • of two lines • of a line and a plane • of two planes		
	Orthogonal projection in 3D space	**Define** the orthogonal projection of a point on a plane **Determine** the coordinates of the orthogonal projection of a point on a plane		

	Distances in 3D space	**Calculate** the distance: • from a point to a line • from a point to a plane • between two planes • between two parallel lines		• Investigate the distance between skew lines. • Investigate spheres in 3D space.
	Angles in 3D space	**Calculate** the angle between: • two vectors • two lines • two planes • a plane and a line **Use** the properties of vectors to study the parallelism: • of two lines • of a line and a plane • of two planes		
		Use a technological tool to investigate all possible intersections		Show the link between equations and a locus in a space.
				• Points, Lines, angles and triangles in Non-Euclidean geometries. • Non-Euclidean geometry and Einstein's theory of general relativity. • Banach-Tarski paradox.

Year 7 (5P)	TOPIC: STATISTICS AND PROBABILITY			
Subtopic	**Content**	**Learning objectives**	**Key contexts, phenomena and activities**	
Bivariate statistics	Visualisation	**Draw** a scatter diagram as a visualisation of a bivariate data set		Explore examples of physics, chemistry, biology and economics: • Relationship between time and speed or time and distance • Relationship between mass and extension of a spring • Relationship between weight and height • Relation of production quantity and price
		Calculate and **plot** the mean point for a scatter diagram		
	Correlation	**Interpret** a scatter diagram in terms of a possible relationship between the variables **Investigate** the idea of a line of best fit (linear regression by eye)		• Investigate examples in which correlation and causality are misused. • Find examples of correlation – causality confusion in media.
	Regression	**Understand** the method of least squares when calculating the least square regression line (regression of y on x) **Understand** the idea of Pearsons' correlation coefficient r for a linear regression model **Calculate** Pearsons' correlation coefficient with a technological tool **Interpret** Pearsons' correlation coefficient in terms of a strong or weak, positive or negative relationship		

		Use a technological tool to: • enter bivariate data • draw a scatter diagram • plot the mean point • draw least square regression line • calculate r		
		Investigate other appropriate regression models (polynomial, logarithmic, exponential), on a scatter diagram by using trial and error, **identify** and **eliminate** outliers where necessary		Coefficient of determination
	Applications	**Apply** correlation and regression to problem situations with real data		Apply this to the pupils' own data set.
		Use a regression model to make interpolations, extrapolations and forecasts		Investigate the limits of extrapolation linked to the fact that a model often only applies on a certain time frame.
Continuous distributions	The notion of a continuous distribution	**Understand** the concept of a continuous random variable and the notion of a continuous distribution in which probabilities correspond to areas under density *Limitation: No theoretical development is necessary* **Compare** discrete and continuous distributions		
	Probability density function	**Understand** the concept of probability density function of a continuous random variable X and its properties: $f(x) \geq 0$ and $\int_{-\infty}^{\infty} f(x)\mathrm{d}x = 1$		

		Use the probability density function to calculate probabilities: $\mathrm{P}(a < \mathrm{X} < b) = \int_a^b f(x)\mathrm{d}x$		Relate to integration theory in calculus.
	Cumulative distribution function	**Understand** and **apply** the cumulative distribution function of a continuous random variable X, defined by: $\mathrm{F}(x) = \mathrm{P}(\mathrm{X} \leq x) = \int_{-\infty}^{x} f(t)\mathrm{d}t$ where f is the probability density function of X		Integration of discontinuous functions.
	Expected value, variance and standard deviation	**Understand** and **interpret** the concept of expected value of a continuous random variable X **Calculate** the expected value $\mathrm{E(X)} = \int_{-\infty}^{\infty} x \cdot f(x)\mathrm{d}x$ where f is the probability density function of X **Understand** and **interpret** the concept of variance of a continuous random variable X **Calculate** the variance $\mathrm{Var(X)} = \int_{-\infty}^{\infty} (x - \mathrm{E(X)})^2 \cdot f(x)\mathrm{d}x$ where f is the probability density function of X **Derive and use** the formula $\mathrm{Var(X)} = \mathrm{E(X^2)} - (\mathrm{E(X)})^2$ **Define** the standard deviation of a continuous random variable X as : $\sigma(\mathrm{X}) = \sqrt{\mathrm{Var(X)}}$		Relate to centre of gravity in physics.
	Modelling with the normal distribution	**Understand** that many data sets can be modelled by a unimodal, symmetric normal distribution, characterised by its expected value and standard deviation		• Discuss the central limit theorem. • Normal approximation to a binomial distribution: recognise the conditions for a normal approximation to a binomial distribution ($npq > 9$) and use this approximation including the continuity correction.

				• Exponential distribution: understand that phenomena that involve the time needed to wait for an event to happen in a memoryless process can be modelled by a negative exponential distribution, know and use the probability density function for an exponentially distributed random variable X, $f(x) = \lambda \mathrm{e}^{-\lambda x}$ for $x \geq 0$. Use integration to prove that $\mathrm{E}(\mathrm{X}) = \frac{1}{\lambda}$ and $\mathrm{Var}(\mathrm{X}) = \frac{1}{\lambda^2}$ Use the examples of radioactive decay or queuing time.
		Calculate for a random variable X following a normal distribution with given mean μ and standard deviation σ: • $\mathrm{P}(\mathrm{X} \leq b)$ • $\mathrm{P}(a \leq \mathrm{X})$ • $\mathrm{P}(a \leq \mathrm{X} \leq b)$ • a if $\mathrm{P}(\mathrm{X} \leq a) = k$ is given		
	Standardisation	**Standardise** a normal distributed random variable X with expected value μ and standard deviation σ as $\mathrm{Y} = \frac{\mathrm{X}-\mu}{\sigma}$. **Determine** values of the mean μ and/or standard deviation σ using the standardised form.		
	Applications and use of technology	**Use** a technological tool to fit a normal distribution curve to a given data set represented graphically and use this to estimate probabilities **Apply** the above in problem situations with real data		

5. Assessment

For each level there are attainment descriptors written by the competences, which give an idea of the level that pupils have to reach. They also give an idea of the kind of assessments that can be done.

With the competences are verbs that give an idea of what kind of assessment can be used to assess that goal. In the table with learning objectives these verbs are used and put bold, so there is a direct link between the competences and the learning objectives.

Assessing content knowledge can be done by written questions where the pupil has to respond on. Partly that can be done by multiple choice but competences as constructing explanations and engaging in argument as well as the key competences as communication and mathematical competence need open questions or other ways of assessing.

An assignment where pupils have to use their factual knowledge to make an article or poster about a (broader) subject can be used to also judge the ability to critically analyse data and use concepts in unfamiliar situations and communicate logically and concisely about the subject.

In Europe, pupils must have some competence in designing and/or engineering (STEM education). So there has to be an assessment that shows the ability in designing and communicating. A design assessment can also show the ability in teamwork.

Pupils have to be able to do an (experimental) inquiry. An (open) inquiry should be part of the assessments. Assessing designing and inquiry can be combined with other subjects or done by one subject, so pupils are not obliged to do too many designing or open inquiry just for assessment at the end of a year.

Digital competence can be assessed by working with spreadsheets, gathering information from internet, measuring data with measuring programs and hardware, modelling theory on the computer and comparing the outcomes of a model with measured data. Do combine this with other assessments where this competence is needed.

Assessment is formative when either formal or informal procedures are used to gather evidence of learning during the learning process and are used to adapt teaching to meet pupil needs. The process permits teachers and pupils to collect information about pupil progress and to suggest adjustments to the teacher's approach to instruction and the pupil's approach to learning.

Assessment is summative when it is used to evaluate pupil learning at the end of the instructional process or of a period of learning. The purpose is to summarise the pupils' achievements and to determine whether, and to what degree, the pupils have demonstrated understanding of that learning.

For all assessment, the marking scale of the European schools shall be used, as described in "Marking system of the European schools: Guidelines for use" (Ref.: 2017-05-D-29-en-7).

S7 final Baccalaureate mark in Mathematics

The S7 5 Period course is not only to be assessed using the written Baccalaureate exam (BAC) and Pre-Baccalaureate exams (preBAC) but also with continuous teacher assessment using the various competences. These continuous, formative assessments do not need to follow the format of the Baccalaureate exam but can vary in type and structure.

The preBAC and BAC on the other hand will be very similar in structure, although only the BAC will be written centrally, while the preBACs will be harmonised within each school. This means that all topics from S6 up to the time of the preBAC could potentially be included and the guidelines for the number and style of questions in the BAC should apply to the preBAC. In requiring the preBAC to mirror the structure of the BAC, as described below, pupils are assured that the preBACs are comparable between European schools, as well as within each school.

The final Baccalaureate written exam will consist of two parts, one part with and one part without a technological tool. Each part will make up 50% of the total 100 marks and will be 120 minutes long. All topics from the S6 and S7 5 Period courses can be examined in the final written Baccalaureate exam.

The overall weighting of the competences will be set by the Mathematics BAC matrix. The weightings are designed to ensure meaningful marks across the range of performances. Performances should be distinguishable not only quantitatively ("more or less of the same") but also qualitatively ("different levels of attainment"). This will make the paper accessible to all pupils while providing more challenging problems to obtain higher marks.

To achieve the highest marks, higher order thinking skills will be required to fully answer the papers. Pupils should be prepared to meet a small number of questions that have an unfamiliar format and require more abstract problem solving and interpretation skills. It is also possible that a single question might require skills and knowledge from several topics, for example probability and sequences. Although all topics in the syllabus will be covered, the proportion of each topic is not fixed. Questions may contain sub-questions. Pupils are encouraged to attempt to solve all questions, so that they can demonstrate a wide variety of skills throughout the exam. Partial solutions can be allowed some marks.

The part without the technological tool will consist of six short response questions and two longer questions that require more extended mathematical thinking. These extended questions could be structured, giving pupils greater guidance, or more open, requiring pupils to develop a suitable strategy for solving the problem.

The part with the technological tool will consist of a smaller number of longer, structured questions that allow pupils to explore a given context in more depth. In general, the level of thinking will increase as a pupil works through the questions. The technological tool will need to be used to fully answer this paper, though this does not exclude the possibility that some questions could be fully answered without the use of the tool.

For all assessment, the marking scale of the European schools shall be used, as described in "*Marking system of the European schools: Guidelines for use*" (Ref.: 2017-05-D-29-en-7).

5.1. Attainment Descriptors

	A	B	C	D	E	F	FX
	(9,0 - 10 Excellent)	(8,0 - 8,9 Very good)	(7,0 - 7,9 Good)	(6,0 - 6,9 Satisfactory)	(5,0 - 5,9 Sufficient)	(3,0 - 4,9 Weak/Failed)	(0 - 2,9 Very weak/Failed)
Knowledge and comprehension	Demonstrates comprehensive knowledge and understanding of mathematical terms, symbols and principles in all areas of the programme	Shows broad knowledge and understanding of mathematical terms, symbols and principles in all areas of the programme	Shows satisfactory knowledge and understanding of mathematical terms, symbols and principles in all areas of the programme	Shows satisfactory knowledge and understanding of mathematical terms, symbols and principles in most areas of the programme	Demonstrates satisfactory knowledge and understanding of straightforward mathematical terms, symbols and principles	Shows partial knowledge and limited understanding of mathematical terms, symbols, and principles	Shows very little knowledge and understanding of mathematical terms, symbols and principles
Methods	Successfully carries out mathematical processes in all areas of the syllabus	Successfully carries out mathematical processes in most areas of the syllabus	Successfully carries out mathematical processes in a variety of contexts	Successfully carries out mathematical processes in straightforward contexts	Carries out mathematical processes in straightforward contexts, but with some errors	Carries out mathematical processes in straightforward contexts, but makes frequent errors	Does not carry out appropriate processes
Problem solving	Translates complex non-routine problems into mathematical symbols and reasons to a correct result; makes and uses connections between different parts of the programme	Translates non-routine problems into mathematical symbols and reasons to a correct result; makes some connections between different parts of the programme	Translates routine problems into mathematical symbols and reasons to a correct result	Translates routine problems into mathematical symbols and reasons to a result	Translates routine problems into mathematical symbols and attempts to reason to a result	N/A	N/A

Interpretation	Draws full and relevant conclusions from information; evaluates reasonableness of results and recognises own errors	Draws relevant conclusions from information, evaluates reasonableness of results and recognises own errors	Draws relevant conclusions from information and attempts to evaluate reasonableness of results	Attempts to draw conclusions from information given, shows some understanding of the reasonableness of results	Attempts to draw conclusions from information and shows limited understanding of the reasonableness of results	Makes little attempt to interpret information	N/A
Communication	Consistently presents reasoning and results in a clear, effective and concise manner, using mathematical terminology and notation correctly	Consistently presents reasoning and results clearly using mathematical terminology and notation correctly	Generally presents reasoning and results clearly using mathematical terminology and notation correctly	Generally presents reasoning and results adequately using mathematical terminology and notation	Generally presents reasoning and results adequately using some mathematical terminology and notation	Attempts to present reasoning and results using mathematical terms	Displays insufficient reasoning and use of mathematical terms
Digital competence	Uses technology appropriately and creatively in a wide range of situations	Uses technology appropriately in a wide range of situations	Uses technology appropriately most of the time	Uses technology satisfactorily most of the time	Uses technology satisfactorily in straightforward situations	Uses technology to a limited extent	Does not use technology satisfactorily

Annex 1: Suggested time frame

For this cycle, the following topics are described with only an estimated number of weeks to be reviewed by the teacher depending on the class.

Note: The designated weeks include assessments, time needed for practice and rehearsal, mathematics projects, school projects, etcetera.

Course	**S6P5**	**S7P5**
Topic	**Weeks**	
Algebra	4 to 5	5
Analysis	14 to 15	8
Geometry	3	8
Statistics and Probability	8	7
Total	**30**	**28**

Annex 2: Sample BAC papers, solutions and matrix

The following pages include:

- Sample BAC without Tool
- Sample BAC without Tool solutions
- Sample BAC with Tool
- Sample BAC with Tool solutions
- Sample BAC Competency Matrix
    - Without Tool
    - With Tool

MATHEMATICS 5 PERIODS
PART A

DATE: DD/MM/YYYY

DURATION OF THE EXAMINATION: 120 minutes

EXAMINATION WITHOUT TECHNOLOGICAL TOOL

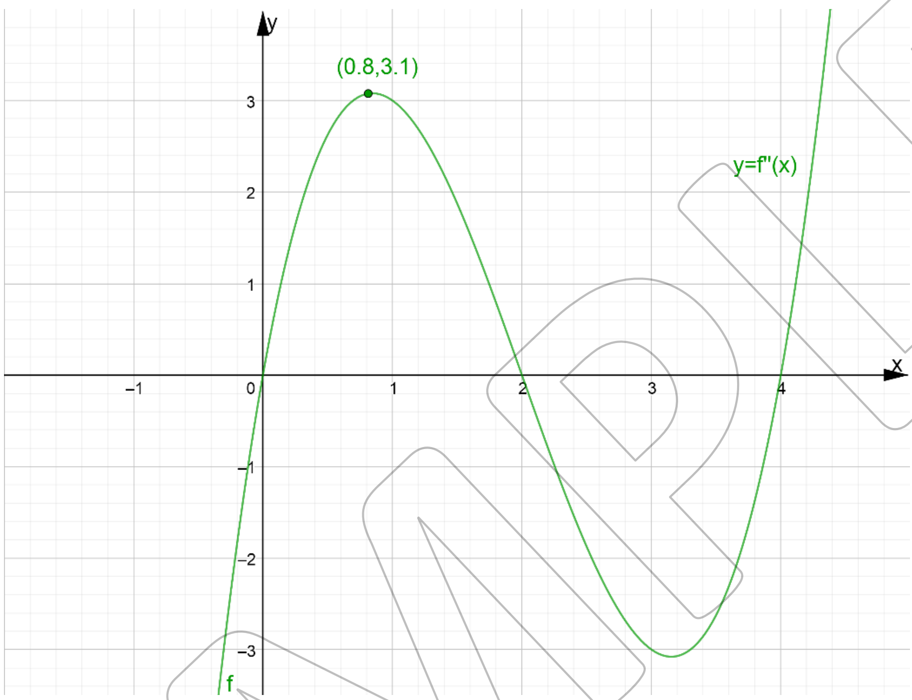
AUTHORISED MATERIAL:

Formula Booklet

Notes:

- As this is a sample paper the cover page is likely to change.
- This sample paper should only be used to see how questions can be created from the syllabus focusing on competences rather than strictly on content.
- The keywords found in the syllabus are highlighted in bold to help the candidate see which competency the question is focusing on and thus helping in answering the question.

PART A		
	Page 1/3	**Marks**
S1	Given the function f, where $f(x) = \ln(3x-2)$, **determine** the equation of the tangent to the graph of f when $x = 1$.	4
S2	**Determine** the complex solutions to the equation: $z^2 = 3i$. **Give** your answers on the form $z = re^{i\theta}$ where $\theta \in]-\pi, +\pi]$.	5
S3	Given the function $f(x) = \frac{2x-1}{x-1}$. Let f^{-1} be the inverse function of f. **Solve** the equation $f^{-1}(x) = 2$.	3
S4	A strictly increasing arithmetic sequence (a_n) and a geometric sequence (b_n) have the same first term, where $a_1 = b_1 = 2$. Additionally, both (a_n) and (b_n) have the same third term. That is $a_3 = b_3$ The sum of the first three terms of the arithmetic sequence is 4 greater than the sum of the first three terms of the geometric sequence. **Determine** the formula for the nth term of both (a_n) and (b_n).	7
S5	A continuous random variable X has a density function given by a formula: $$f(x) = \begin{cases} 0 & , x < 0 \\ a \cdot e^{-ax} & , x \geq 0 \end{cases}$$ We know that $P(X < 1) = \frac{1}{2}$. **Show** that $a = \ln 2$.	5

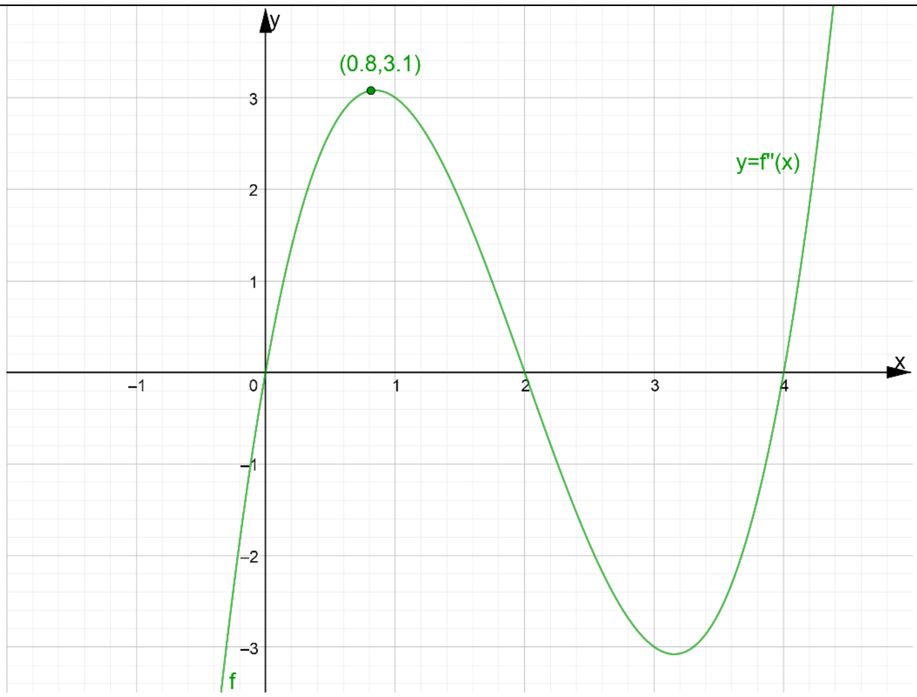
PART A		
	Page 2/3	Marks
S6	Given is the graph of the second derivative f'' of a function (see figure below). **Decide** which of the following statements are true and which are false. **Justify** your answer. a) The graph of f is concave for $-0{,}5 < x < 2$. b) The graph of f has an inflection point at $x = 0$. c) If $f'(0) = 0$, then the graph of f has an inflection point with a horizontal tangent at $x = 0$. 	 2 2 2

PART A		
	Page 3/3	Marks
E1	A drone manufacturer tests new types of drones at a local athletics field. Drone A moves along the path given by the equation: $$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 10 \\ 13 \\ 0 \end{pmatrix} + t\begin{pmatrix} 3 \\ 4 \\ 12 \end{pmatrix}, t \geq 0$$ The time t is in seconds and distance is measured in meters. a) **Find** the position of drone A after 6 seconds.	2
	b) **Determine** how long it will take the drone A to reach the point $(25,33,60)$.	2
	c) **Calculate** the speed of the drone A. Give your answer in a simplest surd form.	2
	d) There is an observer watching drone A from the point $(13,53,0)$. **Calculate** the shortest distance between the drone A and the observer, and the time when it occurs.	3
	Drone B takes off from the point $(9,11,0)$ and moves at 7 m/s in the direction $\begin{pmatrix} 1 \\ 1.5 \\ 3 \end{pmatrix}$. e) **Show** that the equation describing the position of the drone B is: $$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 9 \\ 11 \\ 0 \end{pmatrix} + t\begin{pmatrix} 2 \\ 3 \\ 6 \end{pmatrix}, t \geq 0$$	2
	f) **Find** the point at which the paths of the drones A and B intersect.	2
	g) **Decide** whether the drones will collide at this point. **Justify** your answer.	2
E2	Two players, A and B alternately and independently flip a fair coin. The first player to get a head wins. Assume player A flips first. a) **Write down** the probability that A wins in a first throw. b) **Calculate** the probability that A wins in a third throw. c) **Determine** the probability that A obtains the first head.	5

Part A Answers

	Part A	Points			
		KC	M	PS	I
S1	Given the function f, where $f(x) = \ln(3x-2)$, **determine** the equation of the tangent to the graph of f when $x = 1$.				
	$f(1) = 0$ $f'(1) = 3$ $0 = 3(1) + c$ $c = -3$ $y = 3x - 3$	3	1		
S2	**Determine** the complex solutions to the equation: $z^2 = 3i$. **Give** your answers on the form $z = re^{i\theta}$ where $\theta \in]-\pi, +\pi]$.				
	$z = (3i)^{\frac{1}{2}} = (3cis\left(\frac{\pi}{2}\right))^{\frac{1}{2}} = \sqrt{3}cis\left(\frac{\pi}{4}\right) = \sqrt{3}e^{\frac{i\pi}{4}}$	3	2		
S3	Given the function $f(x) = \frac{2x-1}{x-1}$. Let f^{-1} be the inverse function of f. **Solve** the equation $f^{-1}(x) = 2$.				
	Determine the inverse function: $y = \frac{2x-1}{x-1}$ $y \cdot x - y = 2x + 1$ $y \cdot x - 2x + 1 = y$ $x \cdot (y-2) = y - 1$ $x = \frac{y-1}{y-2}$ exchange x and y: $y = f^{-1}(x) = \frac{x-1}{x-2}$ $f^{-1}(x) = \frac{x-1}{x-2} = 2$ $x - 1 = 2x - 4 \quad \Rightarrow \quad x = 3$		2	1	
S4	A strictly increasing arithmetic sequence (a_n) and a geometric sequence (b_n) have the same first term, where $a_1 = b_1 = 2$. Additionally, both (a_n) and (b_n) have the same third term. That is $a_3 = b_3$ The sum of the first three terms of the arithmetic sequence is 4 greater than the sum of the first three terms of the geometric sequence. **Determine** the formula for the nth term of both (a_n) and (b_n).				
	$a_1 = b_1 = 2$ $a_3 = b_3 \quad \Leftrightarrow \quad a_1 + 2d = b_1 \cdot q^2 \quad \Leftrightarrow \quad 2 + 2d = 2 \cdot q^2 \quad \|:2$ $1 + d = q^2 \quad \|-1$ $d = q^2 - 1 \quad (I)$ $s_{a,3} = s_{b,3} + 4 \quad \Leftrightarrow \quad 3a_1 + 3d = b_1\left(1 + q + q^2\right) + 4$ $\Leftrightarrow \quad 6 + 3d = 2\left(1 + q + q^2\right) + 4 \quad \|-4$ $2 + 3d = 2 + 2q + 2q^2 \quad (II)$	2	2	3	

	(I) in (II): $2+3(q^2-1)=2+2q+q^2$ $2+3q^2-3=2+2q+q^2 \quad \vert -2-2q-2q^2$ $q^2-2q-3=0$ pq-formula: $q=1\pm\sqrt{1+3} \quad \Rightarrow \quad q_1=3, \qquad q_2=-1$ $\Rightarrow \quad d_1=3^2-1=8, \qquad d_2=(-1)^2-1=0$ d_2 can be excluded, as the sequence (a_n) is strictly increasing according to the precondition. $\Rightarrow \begin{matrix}(a_n)=2+(n-1)\cdot 8=8n-6 \\ (b_n)=2\cdot 3^{n-1}\end{matrix}$				
S5	A continuous random variable *X* has a density function given by a formula: $$f(x)=\begin{cases}0 & ,x<0 \\ a\cdot e^{-ax} & ,x\geq 0\end{cases}$$ We know that $P(X<1)=\frac{1}{2}$. **Show** that $a=\ln 2$.				
	$\int_0^1 ae^{-ax}dx={}_0^1[-e^{-ax}]=-e^{-a}+1$ $-e^{-a}+1=\frac{1}{2}$ $e^{-a}=\frac{1}{2}$ $e^{a}=2$ $a=\ln(2)$		2	3	
S6	Given is the graph of the second derivative f'' of a function (see figure below). **Decide** which of the following statements are true and which are false. **Justify** your answer. a) The graph of f is concave for $-0,5<x<2$. b) The graph of f has an inflection point at $x=0$. c) If $f'(0)=0$, then the graph of f has an inflection point with a horizontal tangent at $x=0$.				

					
	a) False. For -0.5<x<2 f''(x) assumes values greater than and less than zero. b) True. f'' is equal to zero at x=2 with change of sign from + to -. c) True. If f'(0)=0, f''(0)=0, and second derivative changes its sign at x=0,then we have a stationary point of inflection.		 1 2	2 1	
E1	A drone manufacturer tests new types of drones at a local athletics field. Drone A moves along the path given by the equation: $$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 10 \\ 13 \\ 0 \end{pmatrix} + t\begin{pmatrix} 3 \\ 4 \\ 12 \end{pmatrix}, t \geq 0$$ The time t is in seconds and distance is measured in meters. a) **Find** the position of drone A after 6 seconds. b) **Determine** how long it will take the drone A to reach the point $(25,33,60)$. c) **Calculate** the speed of the drone A. Give your answer in a simplest surd form. d) There is an observer watching drone A from the point $(13,53,0)$. **Calculate** the shortest distance between the drone A and the observer, and the time when it occurs. Drone B takes off from the point $(9,11,0)$ and moves at 7 m/s in the direction $\begin{pmatrix} 1 \\ 1.5 \\ 3 \end{pmatrix}$. e) **Show** that the equation describing the position of the drone B is: $$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 9 \\ 11 \\ 0 \end{pmatrix} + t\begin{pmatrix} 2 \\ 3 \\ 6 \end{pmatrix}, t \geq 0$$ f) **Find** the point at which the paths of the drones A and B intersect. g) **Decide** whether the drones will collide at this point. **Justify** your answer.				
	a) t=6, so (28,37,72) b) $\begin{pmatrix} 25\cdot \\ 33 \\ 60 \end{pmatrix} = \begin{pmatrix} 10 \\ 13 \\ 0 \end{pmatrix} + t\begin{pmatrix} 3 \\ 4 \\ 12 \end{pmatrix}$, t=5 c) $\sqrt{3^2+4^2+12^2} = \sqrt{169} = 13\ m/s$	1 1	1 1 1	 1	

	d) $\begin{pmatrix} -3+3t \\ -40+4t \\ 12t \end{pmatrix} \circ \begin{pmatrix} 3 \\ 4 \\ 12 \end{pmatrix} = 0, so\ t = 1$ and $\left\lvert \begin{pmatrix} 0 \\ -36 \\ 12 \end{pmatrix} \right\rvert = \sqrt{1440} = 12\sqrt{10}$ e) $\left\lvert \begin{pmatrix} 1 \\ 1.5 \\ 3 \end{pmatrix} \right\rvert = \sqrt{12.25} = 3.5 \quad but\ the\ speed\ of\ the\ drone\ B\ is\ two\ times\ bigger,$ $so\ the\ velocity\ vector\ of\ the\ drone\ B\ must\ be\ \begin{pmatrix} 2 \\ 3 \\ 6 \end{pmatrix}$ f) $\begin{pmatrix} 10 \\ 13 \\ 0 \end{pmatrix} + t_A \begin{pmatrix} 3 \\ 4 \\ 12 \end{pmatrix} = \begin{pmatrix} 9 \\ 11 \\ 0 \end{pmatrix} + t_B \begin{pmatrix} 2 \\ 3 \\ 6 \end{pmatrix}$, point of intersection *(13,17,12)* g) $No\ because\ t_A = 1s\ and\ t_B = 2s$		2 2	1 1 2	1
E2	Two players, A and B alternately and independently flip a fair coin. The first player to get a head wins. Assume player A flips first. a) **Write down** the probability that A wins in a first throw. b) **Calculate** the probability that A wins in a third throw. c) **Determine** the probability that A obtains the first head.				
	a) $\frac{1}{2}$ b) $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$ c) Player one can win at the first, the third, the fifth throw., so P(A)=$\frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \ldots \ldots \ldots$ P(A)=$\frac{1}{2} + \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^5_{\ldots\ldots}$ It leads the infinite geometric sequence with $a_1 = \frac{1}{2}$ and $q = \frac{1}{4}$, so $$s_\infty = \frac{a}{1-q} = \frac{\frac{1}{2}}{\frac{3}{4}} = \frac{2}{3}$$			3	2

MATHEMATICS 5 PERIODS
PART B

DATE: DD/MM/YYYY

DURATION OF THE EXAMINATION: 120 minutes

EXAMINATION WITH TECHNOLOGICAL TOOL

AUTHORISED MATERIAL:

Technological tool

Formula Booklet

Notes:

- As this is a sample paper the cover page is likely to change.
- This sample paper should only be used to see how questions can be created from the syllabus focusing on competences rather than strictly on content.
- The keywords found in the syllabus are highlighted in bold to help the candidate see which competency the question is focusing on and thus helping in answering the question.

PART B

Question 1/4 | Marks

Tom and Simon play a board game. Each time Tom manages to move his piece around the board he gets 5 points. Each time Simon manages to move his piece around the board he gets 10% of the previous amount. They both start with 10 points.

a) **Calculate** Tom's total score after moving around the board 20 times. 2

b) **Write** in terms of n the formula $\boldsymbol{T(n)}$ for Tom's score after $\boldsymbol{n}$ moves around the board. 2

c) If you know that Simon's score after n moves around the board could be modelled with a geometric sequence, **explain** the use of the formula: 2

$$\boldsymbol{S(n) = 11 \cdot 1.1^{n-1}}$$

d) Simon and Tom have been around the board the same number of times. Simon's score has just moved ahead of Tom's. 3

Find how many times have they been around the board.

Tom challenges Simon to a dice game. Two fair six-sided die are rolled and the sum of scores is noted. For a sum less than 6 Simon receives 10 cents, for a sum between 6 and 9 Simon loses 5 cents, and for the sum bigger or equal 10 Simon receives 30 cents. The winnings are governed by the probability distribution shown below, where the random variable N is the sum of scores.

N	$\boldsymbol{n < 6}$	$\boldsymbol{6 \le n \le 9}$	$\boldsymbol{n \ge 10}$
Winnings n	10 cents	−5 cents	30 cents
$\boldsymbol{P(N = n)}$	a	$\boldsymbol{\frac{20}{36}}$	b

e) **Show**, that $\boldsymbol{a} = \boldsymbol{\frac{10}{36}}$ et $\boldsymbol{b} = \boldsymbol{\frac{6}{36}}$. 2

f) **Calculate** the expected value of Simon's winnings in this game and comment if it is worth Simon playing. 2

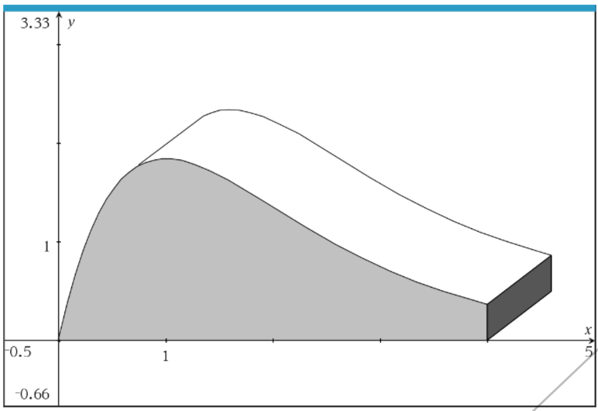
g) A game is said to be fair if the expected value is 0. 2

Determine how many cents should be lost for the sum between 6 and 9 to make this game fair.

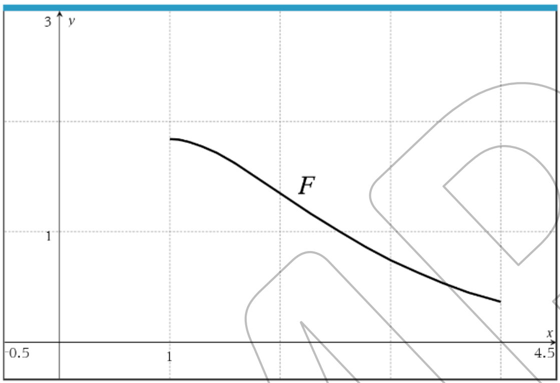
PART B

Question 2/4 — Marks

A kids' play area manufacturer wants to offer its customers a new model of slide. They create a diagram of the proposed slide in an oblique projection:



The profile of this slide is measured in meters and can be modeled by the function $F(x) = (ax - b)e^{-x}$, for $1 \leq x \leq 4$, where a and b are two parameters. The function F was drawn below.



a) It is planned that the tangent to the function F at the point where $x = 1$ would be horizontal. [3 marks]

Determine the value of the parameter b.

b) It is also planned that the top of the slide will be at 1.85 meters. [2 marks]

Determine the value of the parameter a.

The profile of the wall is finally modeled by $F(x) = 5x \cdot e^{-x}$.

c) **Show** that the total area of each side wall, shaded grey on the diagram is equal to $5 - \frac{25}{e^4}$ m². [2 marks]

d) **Determine** the point on the slide where the gradient is greatest. [3 marks]

PART B

Question 3/4	Marks
Optical smoke detectors contain a photocell as an important component. A factory produces photocells for this purpose. A controller automatically checks photocells and rejects those that are faulty. On average he is 86% accurate. However, the accuracy of the controller is found to vary - sometimes he detects a higher percentage of faulty photocells and sometimes a lower percentage. The controller's accuracy is found to be modelled by a normal distribution with a standard deviation of 5%.	
a) **Find** the probability that the controller is less than 85% accurate.	1
b) $\frac{9}{10}$ of the time the controller is less than $x\%$ accurate. **Determine** x.	2
c) Given that, on a particular day, the controller is less than 90% accurate, **find** the probability that he is more than 85% accurate.	2
Two types of optical smoke detector are being tested for reliability. The higher the probability of an alarm being triggered the more reliable it is.	
Type A contains a single photocell and is triggered when this photocell is activated.	
Type B contains three photocells and is triggered if at least two of the three photocells are activated.	
The probability of a photocell being activated in the presence of smoke is p. The probability of both types of alarm being triggered is calculated for different values of p.	
$P(A_p)$ is the probability of type A being triggered when the probability is p,	
$P(B_p)$ is the probability of type B being triggered when the probability is p.	
d) **Complete** the table below.	4

p	0.3	0.5	0.7
$P(A_p)$	0.3	0.5	0.7
$P(B_p)$			
More reliable type			

e) **Determine** for what value of p does type B become more reliable than type A.	2
f) **Show** that, in terms of p, $P(A_p) = p$ and $P(B_p) = -2p^3 + 3p^2$.	4
g) **Explain** the meaning of the following function R in relation to the context of the question. **Explain** what is calculated in lines (1) to (3) and **interpret** the result.	3

$$R: p \longmapsto R(p) = -2p^3 + 3p^2 - p$$

$$(1) \quad R'(p) = -6p^2 + 6p - 1$$

$$(2) \quad R'(p) = 0 \implies p_1 \approx 0{,}79$$

$$(3) \quad R''(p_1) < 0$$

PART B	
Question 4/4	**Marks**
Given are the plane $E: 2x_1 - x_2 + 3x_3 = 5$ and for each $a \in \mathbb{R}$ a straight line: $$g_a: \vec{x} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} + t \cdot \begin{pmatrix} 1 \\ a \\ 2 \end{pmatrix}$$	
a) **Determine** the coordinates of the intersection of the straight line g_a with the plane E in terms of a.	4
b) **Find** for which value of a is there no solution. **Interpret** the result geometrically.	3

Part B Answers

<table>
<tr><td rowspan="2"></td><td rowspan="2">Part B</td><td colspan="4">Points</td></tr>
<tr><td>KC</td><td>M</td><td>PS</td><td>I</td></tr>
<tr><td>1</td><td colspan="5">Tom and Simon play a board game. Each time Tom manages to move his piece around the board he gets 5 points. Each time Simon manages to move his piece around the board he gets 10% of the previous amount. They both start with 10 points.
a) Calculate Tom’s total score after moving around the board 20 times.
b) Write in terms of n the formula $\boldsymbol{T(n)}$ for Tom’s score after $\boldsymbol{n}$ moves around the board.
c) If you know that Simon’s score after n moves around the board could be modelled with a geometric sequence, explain the use of the formula:
$$\boldsymbol{S(n) = 11 \cdot 1.1^{n-1}}$$
d) Simon and Tom have been around the board the same number of times. Simon’s score has just moved ahead of Tom’s.
Find how many times have they been around the board.

Tom challenges Simon to a dice game. Two fair six-sided die are rolled and the sum of scores is noted. For a sum less than 6 Simon receives 10 cents, for a sum between 6 and 9 Simon loses 5 cents, and for the sum bigger or equal 10 Simon receives 30 cents. The winnings are governed by the probability distribution shown below, where the random variable N is the sum of scores.
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<tr><td>Winnings n</td><td>10 cents</td><td>−5 cents</td><td>30 cents</td></tr>
<tr><td>$\boldsymbol{P(N = n)}$</td><td>a</td><td>$\boldsymbol{\frac{20}{36}}$</td><td>$b$</td></tr>
</table>
e) Show, that $\boldsymbol{a = \frac{10}{36}}$ et $\boldsymbol{b = \frac{6}{36}}$.
f) Calculate the expected value of Simon’s winnings in this game and comment if it is worth Simon playing.
g) A game is said to be fair if the expected value is 0.
Determine how many cents should be lost for the sum between 6 and 9 to make this game fair.</td></tr>
<tr><td></td><td>a) Calculate Tom’s score after moving around the board 20 times.
$c = 15 + 19 \cdot 5$
$\rightarrow 110$</td><td>2</td><td></td><td></td><td></td></tr>
<tr><td></td><td>b) Write in terms of n the formula T(n) for Tom’s score after n moves around the board.
T(n)=5n+10</td><td>1</td><td>1</td><td></td><td></td></tr>
<tr><td></td><td>c)</td><td></td><td>2</td><td></td><td></td></tr>
<tr><td></td><td>d) Sketch the graph of T(n) and S(n) in the same coordinate system.</td><td></td><td>2</td><td>1</td><td></td></tr>
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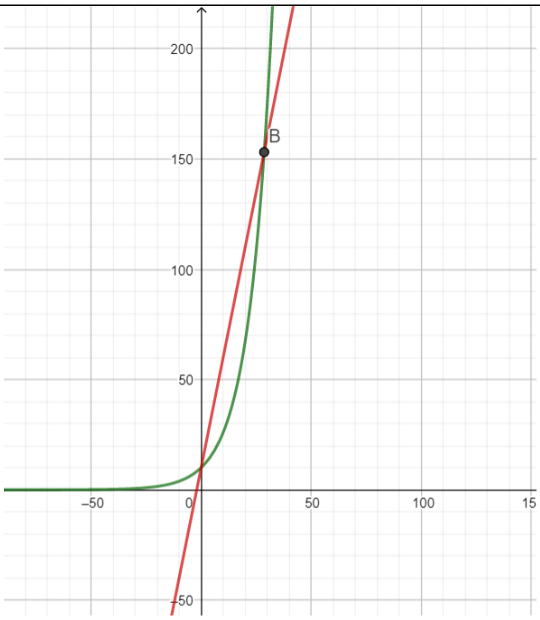

What is the least number of moves around the board for the Simon's score to exceed the Tom's score.
B = Intersect(S, T, (28.63, 153.16))
→ (28.63, 153.16)
Answer: 29 moves
e)
$$\boldsymbol{a = P(X=2) + P(X=3) + P(X=4) + P(X=5)}$$
$$= 1/36 + 2/36 + 3/36 + 4/36 = 10/36$$
$$\boldsymbol{b = 1 - 10/36 - 20/36 = 6/36}$$
f) E(X)= $\frac{\mathbf{10}}{\mathbf{36}} \times 10 + \frac{\mathbf{20}}{\mathbf{36}} \times (-5) + \frac{\mathbf{6}}{\mathbf{36}} \times 30 = 5$
Yes, it is worth playing, because the expected value is a positive number.
g) E(X)= $\frac{\mathbf{10}}{\mathbf{36}} \times 10 + \frac{\mathbf{20}}{\mathbf{36}} \times x + \frac{\mathbf{6}}{\mathbf{36}} \times 30 = 0$, x=-14
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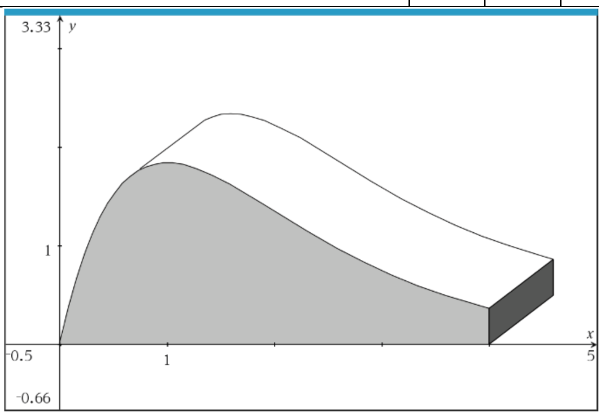
A kids' play area manufacturer wants to offer its customers a new model of slide. They create a diagram of the proposed slide in an oblique projection:


The profile of this slide is measured in meters and can be modeled by the function $F(x) = (ax - b)e^{-x}$, for $1 \leq x \leq 4$, where a and b are two parameters. The function F was drawn below.
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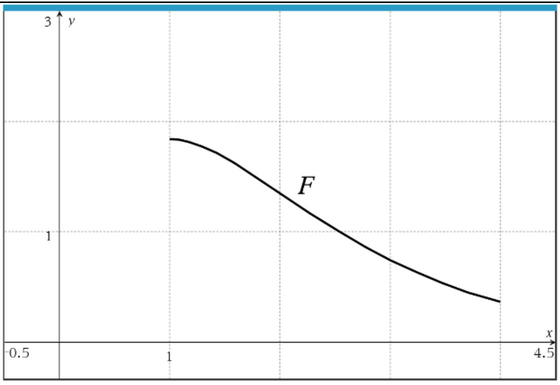

a) It is planned that the tangent to the function F at the point where $x = 1$ would be horizontal.
Determine the value of the parameter b.
b) It is also planned that the top of the slide will be at 1.85 meters.
Determine the value of the parameter a.
The profile of the wall is finally modeled by $F(x) = 5x \cdot e^{-x}$.
c) **Show** that the total area of each side wall, shaded grey on the diagram is equal to $5 - \frac{25}{e^4}$ m².
d) **Determine** the point on the slide where the gradient is greatest.
</td></tr>
<tr><td></td><td>
a/b)
1 Derivative$((a\,x - b)\,e^{-x}, x, 1)$
$\rightarrow\ \mathbf{a\,e^{-x} + b\,e^{-x} - a\,x\,e^{-x}}$
2 Solve$(a\,e^{-1} + b\,e^{-1} - a\,e^{-1} = 0, b)$
$\rightarrow\ \{\mathbf{b = 0}\}$
3 Solve$(a\,e^{-1} = 1.85, a)$
$\rightarrow\ \left\{\mathbf{a = \frac{37}{20}\,e}\right\}$

c) **Show** that the total area of each side wall, shaded grey on the diagram is equal to $5 - \frac{25}{e^4}$ m².

$\begin{cases} u = 5x, & v' = e^{-x} \\ u' = 5, & v = -e^{-x} \end{cases}$
$\int 5xe^{-x}\,dx = -5xe^{-x} + 5\int e^{-x}\,dx$
$-5xe^{-x} - 5e^{-x} + c$
$\int_0^4 5xe^{-x}\,dx = [-5xe^{-x} - 5e^{-x}]_0^4 =$
$(-5 \cdot 4e^{-4} - 5e^{-4}) - (-510e^0 - 5e^0) =$
$-\frac{20}{e^4} - \frac{5}{e^4} + 5=$
$(5 \quad -\frac{25}{e^4})$ m²
d) x=2
A = Min(f1′(x), 0, 4)
→ (2, -0.68)
</td>
<td>2

1

2</td>
<td>1

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<td></td></tr>
<tr><td>3</td><td colspan="5">Optical smoke detectors contain a photocell as an important component. A factory produces</td></tr>
</table>

photocells for this purpose. A controller automatically checks photocells and rejects those that are faulty. On average he is 86% accurate. However, the accuracy of the controller is found to vary - sometimes he detects a higher percentage of faulty photocells and sometimes a lower percentage. The controller's accuracy is found to be modelled by a normal distribution with a standard deviation of 5%.

a) **Find** the probability that the controller is less than 85% accurate.

b) $\frac{9}{10}$ of the time the controller is less than $x\%$ accurate. **Determine** x.

c) Given that, on a particular day, the controller is less than 90% accurate, **find** the probability that he is more than 85% accurate.

Two types of optical smoke detector are being tested for reliability. The higher the probability of an alarm being triggered the more reliable it is.

Type A contains a single photocell and is triggered when this photocell is activated.

Type B contains three photocells and is triggered if at least two of the three photocells are activated.

The probability of a photocell being activated in the presence of smoke is p. The probability of both types of alarm being triggered is calculated for different values of p.

$P(A_p)$ is the probability of type A being triggered when the probability is p,

$P(B_p)$ is the probability of type B being triggered when the probability is p.

d) **Complete** the table below.

p	0.3	0.5	0.7
$P(A_p)$	0.3	0.5	0.7
$P(B_p)$			
More reliable type			

e) **Determine** for what value of p does type B become more reliable than type A.

f) **Show** that, in terms of p, $P(A_p) = p$ and $P(B_p) = -2p^3 + 3p^2$.

g) **Explain** the meaning of the following function R in relation to the context of the question. **Explain** what is calculated in lines (1) to (3) and **interpret** the result.

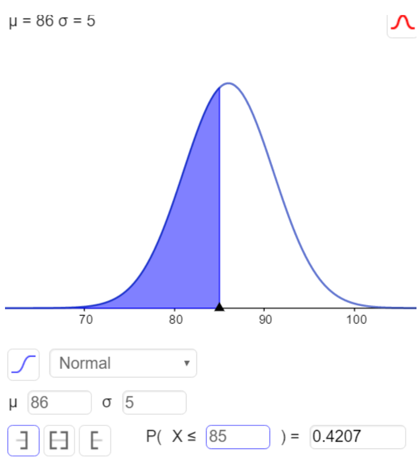
$$R: p \longmapsto R(p) = -2p^3 + 3p^2 - p$$

$$(1) \quad R'(p) = -6p^2 + 6p - 1$$

$$(2) \quad R'(p) = 0 \implies p_1 \approx 0{,}79$$

$$(3) \quad R''(p_1) < 0$$

a)



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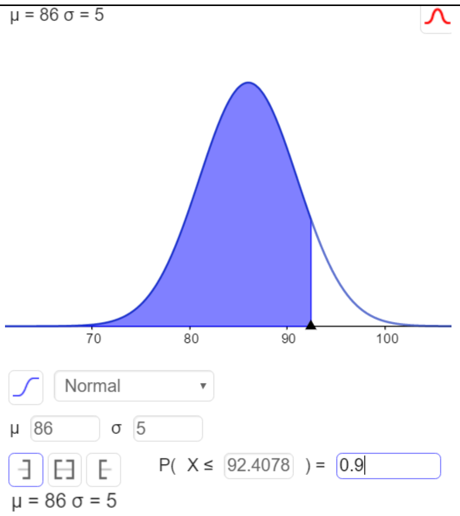
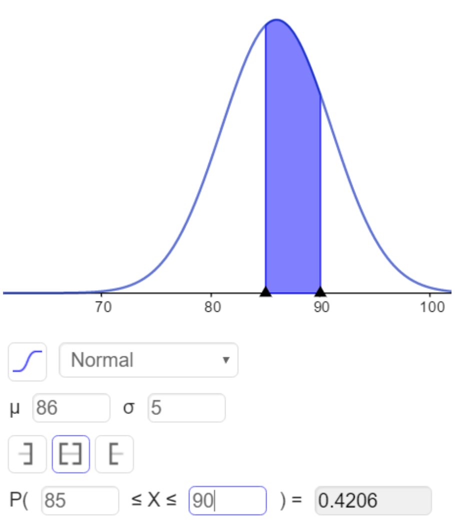
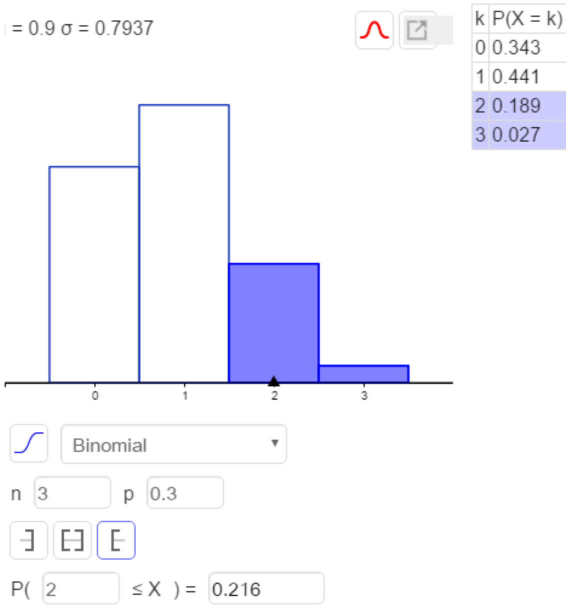
1 1

	b) 	1	1		
	c) 				
	d) The random variable y: "Number of photocells showing a reaction" is assumed to be binomially distributed with n = 3 and p. $P(Y \geq 2) = P(Y = 2) + P(Y = 3)$ $= \binom{3}{2} \cdot p^2 \cdot (1-p)^1 + \binom{3}{3} \cdot p^3$ $\Longrightarrow P(0{,}3) = 0{,}216;\ \ P(0{,}5) = 0{,}500,$ $P(0{,}7) = 0{,}784$ <u>GeoGebra:</u> (shown for P(0,3) only)		1	3	



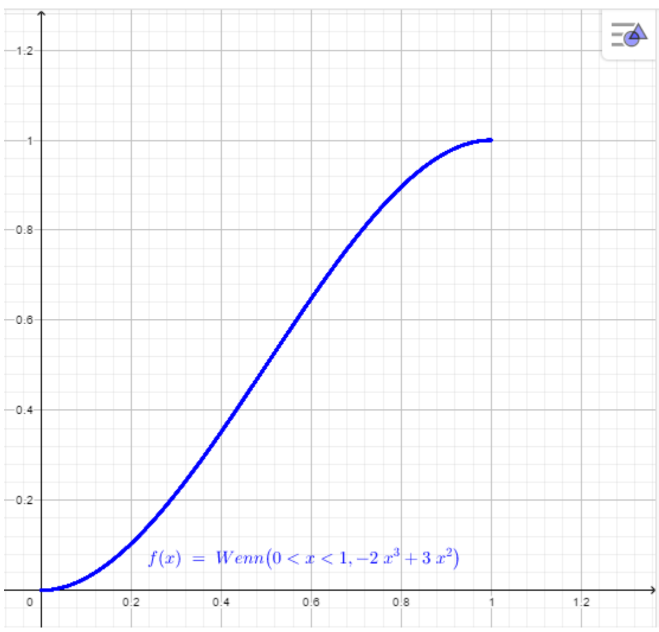
2

1 3

e) The results show that the reliability of smoke detectors is improved if $p > 0.5$, but if $p < 0.5$, the probability of triggering an alarm is reduced by this measure, thus worsening the reliability. If $p = 0.5$, the reliability of an alarm remains consistently good.

f) $P(p) = P(Y \geq 2) = \binom{3}{2} \cdot p^2 \cdot (1-p)^1 + \binom{3}{3} \cdot p^3$

$$= \frac{3!}{2!\,(3-2)!}(p^2 - p^3) + \frac{3!}{3!\,(3-3)!} \cdot p^3$$
$$= 3 \cdot (p^2 - p^3) + 1 \cdot p^3$$
$$= 3p^2 - 3p^3 + p^3 = -2p^3 + 3p^2$$



g) The function R is the difference function of P and p. R indicates for which p the reliability P(p) with three photocells is greater than the reliability p of a single photocell. 1 2

Equation (1): The derivative of R is determined.
Equation (2): The zero of the derivative of the function R is

	calculated. It is located at the position $p_1 \approx 0{,}79$. Equation (3): The sign of the second derivative function at position p_1 is checked. R'' has a negative sign there. In this way, an extremum is calculated for the difference function R, namely a maximum, since $R''(p)$ is negative. If $p \approx 0.79$ is chosen, the difference function $R(p)$ has the highest value, i.e. the reliability of the message is greatest here.				
4	Given are the plane $E: 2x_1 - x_2 + 3x_3 = 5$ and for each $a \in \mathbb{R}$ a straight line: $g_a: \vec{x} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} + t \cdot \begin{pmatrix} 1 \\ a \\ 2 \end{pmatrix}$ a) **Determine** the coordinates of the intersection of the straight line g_a with the plane E in terms of a. b) **Find** for which value of a is there no solution. **Interpret** the result geometrically.				
	a) g_a in E: $2t - (1 + ta) + 3(1 + 2t) = 5$ $2t - 1 - ta + 3 + 6t = 5$ $(8 - a) \cdot t = 3$ $t = \frac{3}{8-a}$ for $a \neq 8$ $\Rightarrow S_a\left(\frac{3}{8-a} \middle\| 1 + \frac{3a}{8-a} \middle\| 1 + \frac{6}{8-a}\right)$		2	2	
	b) There is no solution for $a = 8$. In this case the direction vector of the straight line g_8 is perpendicular to the normal vector of the plane. $\begin{pmatrix} 1 \\ 8 \\ 2 \end{pmatrix} \circ \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} = 0$ $\Rightarrow$ The line and the plane are parallel. (a) g_a in E: Löse(2 (0 + λ) − (1 + λ A) + 3 (1 + 2 λ) = 5, λ) $\rightarrow \left\{\lambda = \frac{-3}{A-8}\right\}$ Intersection point Sa: Vektor((0 + λ, 1 + λ A, 1 + λ · 2)) Ersetze: $\begin{pmatrix} \frac{3}{-A+8} \\ 3 \cdot \frac{A}{-A+8} + 1 \\ 1 + \frac{6}{-A+8} \end{pmatrix}$ No solution for A=8, as the denominator becomes 0. Skalarprodukt({1, 8, 2}, {2, −1, 3}) $\rightarrow$ 0 Geometric interpretation: line and plane are parallel			2	1

SAMPLE EUROPEAN BACCALAUREATE - Maths Competency Matrix

Question Topic	Question	Learning Objective (specific syllabus reference(s))	Paper-specific Marking Scheme				
			Knowledge and Comprehension	Methods	Problem Solvng	Interpretation	Σ
Without Technology							
Analysis	S1	**Calculate** the tangent at a point	3,0	1,0			4,0
Algebra	S2	**Solve** equations of the type $z^n = a$ with $a \in \mathbb{C}$, $n \in \mathbb{N}\backslash\{0;1\}$ **Convert** from one form of complex number to another	3,0	2,0			5,0
Analysis	S3	**Show** an understanding of inverse functions, and of the connections between the graphs of inverse functions		2,0	1,0		3,0
Algebra	S4	**Solve** problems involving the properties of arithmetic and geometric sequences	2,0	2,0	3,0		7,0
Probability	S5	**Know** and **apply** the cumulative distribution function of a continuous random variable X		2,0	3,0	0,0	5,0
Analysis	S6a	**Apply** the concepts differentiation			2,0		2,0
Analysis	S6b	**Apply** the concepts differentiation		1,0	1,0		2,0
Analysis	S6c	**Apply** the concepts differentiation		2,0			2,0
Geometry	E1a	**Use** vector equation of a line	1,0	1,0			2,0
Geometry	E1b	**Use** vector equation of a line	1,0	1,0			2,0
Geometry	E1c	**Find** the velocity vector of a moving object		1,0	1,0		2,0
Geometry	E1d	**Calculate** the distance from a point to a line		2,0	1,0		3,0
Geometry	E1z	**Know** vector equation of a line			1,0	1,0	2,0
Geometry	E1f	**Calculate** the intersection of two lines		2,0			2,0
Geometry	E1g	**Interpret** your answer				2,0	2,0
Probability / Algebra	E2	**Determine** and **apply** a probability density function. **Define** the nth term of a sequence. **Calculate** the limit of a sequence			3,0	2,0	5,0
		S	**10,0**	**19,0**	**16,0**	**5,0**	**50,0**
		%	**20,0**	**38,0**	**32,0**	**10,0**	
		Guideline:	12,5	20,0	15,0	2,5	50,0
		%	25,0	40,0	30,0	5,0	
		Tolerance (Points):	2,0	3,0	2,0	2,0	

With Technology							
Sequences	1a	Solve problems involving the properties of arithmetic and geometric sequences	2,0				2,0
Sequences	1b	**Give** definitions of arithmetic and geometric sequences	1,0	1,0			2,0
Sequences	1c	**Give** definitions of arithmetic and geometric sequences		2,0			2,0
Sequences	1d	**Solve** problems involving the properties of arithmetic and geometric sequences		2,0	1,0		3,0
Probability	1e	**Calculate** probability for simple cases	1,0	1,0			2,0
Probability	1f	**Understand, interpret** and **calculate** the expected value	1,0	1,0			2,0
Probability	1g	**Understand, interpret** and **calculate** the expected value		1,0	1,0		2,0
Analysis	2a	**For a tangent to a curve at a given point, calculate** its gradient and/or equation		2,0	1,0		3,0
Analysis	2b	**Apply** the concepts of limits and differentiation	1,0	1,0			2,0
Analysis	2c	**Know** the concept of an integral on a closed interval $[a, b]$, with the integral interpreted graphically as an area	2,0				2,0
Analysis	2d	**Calculate** first and second derivative functions and **interpret** their relevance		1,0	2,0		3,0
Probability	3a	**Calculate** with the normal distribution	1,0				1,0
Probability	3b	**Calculate** with the normal distribution	1,0	1,0			2,0
Probability	3c	**Calculate** with the normal distribution	1,0	1,0			2,0
Probability	3d	**Calculate** probabilities with a binomial distribution		1,0	3,0		4,0
Probability	3e	**Calculate** probabilities with a binomial distribution			2,0		2,0
Probability	3f	**Use** the Bernoulli formula		1,0	3,0		4,0
Probability / Analysis	3g	**Apply** differentiation to optimization problems			1,0	2,0	3,0
Geometry	4a	**Calculate** the intersection of a line and a plane		2,0	2,0		4,0
Geometry	4b	**Know** and **use** vector definition of parallelism of lines and planes			2,0	1,0	3,0
		S	**11,0**	**18,0**	**18,0**	**3,0**	**50,0**
		%	**22,0**	**36,0**	**36,0**	**6,0**	
		Guideline:	12,5	20,0	15,0	2,5	50,0
		%	25,0	40,0	30,0	5,0	
		Tolerance (Points):	2,0	3,0	2,0	2,0	

Total					
S	**21,0**	**37,0**	**34,0**	**8,0**	**100,0**
%	**21,0**	**37,0**	**34,0**	**8,0**	
Guideline:	25,0	40,0	30,0	5,0	100,0
Tolerance (Points):	4,0	6,0	4,0	4,0	